Kelly Criterion

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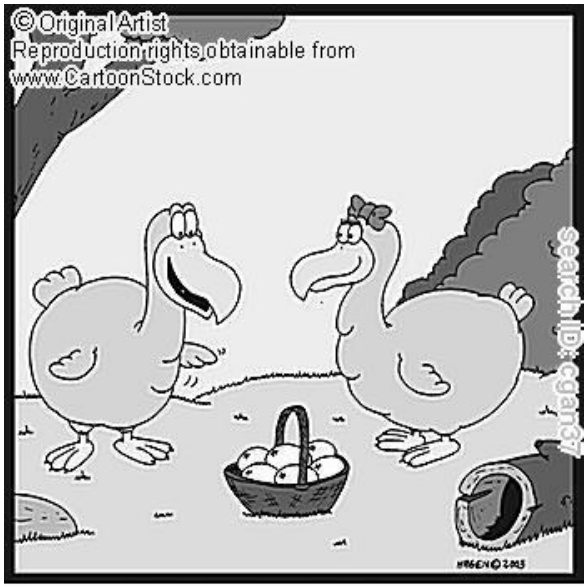
Diversification

- Is diversification good?
- Is it always required?



Simulation by Elton et al

Number of Stocks in Portfolio	Average Standard Deviation of Annual Portfolio Returns	Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.24%	1.00
2	37.36	0.76
4	29.69	0.60
6	26.64	0.54
8	24.98	0.51
10	23.93	0.49
20	21.68	0.44
30	20.87	0.42
40	20.46	0.42
50	20.20	0.41
400	19.29	0.39
500	19.27	0.39
1000	19.21	0.39

Warren Buffet

- I don’t agree
- Sometimes we are better off not diversifying at all
- Put 1/3 rd of its assets in Coca-Cola in the 70’s
- If you have 50 stocks can you like all the stocks equally?

George Soros

- George Soros: Invested $10 Billion in a single trade to break the BOE
- Again risked almost all its firm assets
- Didn’t Diversify
- What's Happening?

Other Examples

- James H. Simon
- Edward O. Throp

What's Happening?

- They are all Kelly Bettor
- William T Ziemba did a detailed analysis of these investors and found them to be Kelly bettors
- Fortunate enough to study under him;)

Growth of Assets

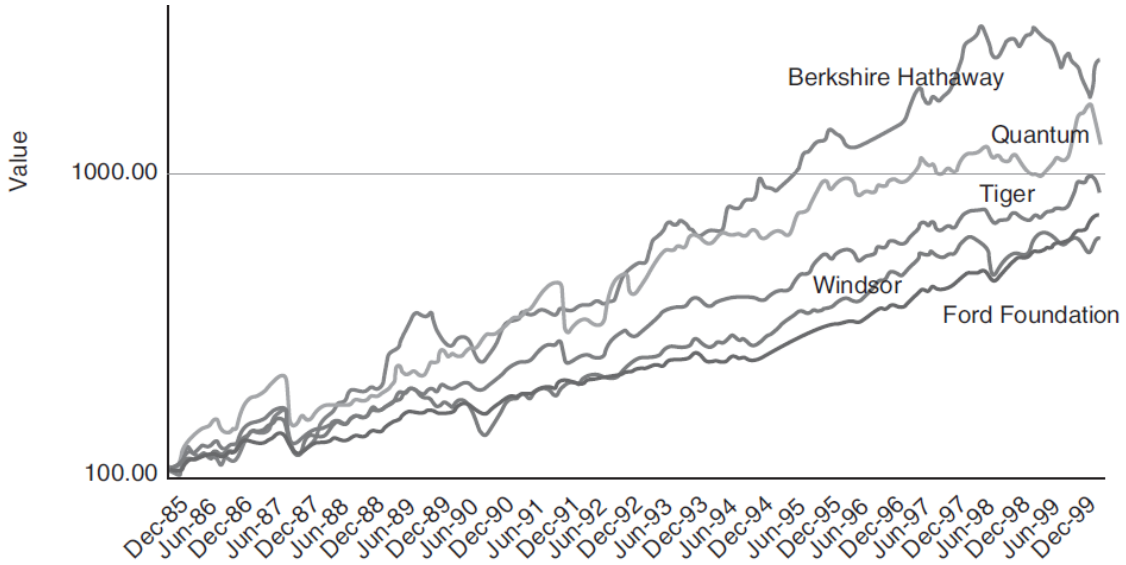


Figure 4.1 Growth of assets, log scale, various high performing funds, 1985–2000. *Source*: Ziemba (2003).

What is Kelly Criterion **?**

- *Probability of Winning trades=p*
- *Probability of Losing trades= q*
- *b= Average gain / average loss*

$$f^* = \frac{bp - q}{b}$$

Explanation

- What is the probability of getting 1 in a roll of dice?
- If 1 comes you win
- Otherwise you lose

Explanation

- 1/6
- So probability of winning=1/6
- Probability of losing= 5/6

Explanation

- Now lets say, I tell you if 1 comes then I will give you 6 dollars otherwise you will lose 2 dollars
- 1= +6
- 2,3,4,5 or 6 = -2
- Should you play this game?

Explanation

- P=1/6
- Q= 5/6
- B= 6/2= 3:1
- Kelly Fraction = [(1/6*3)-(5/6)]/3=-1/9
- Bad trade
- Negative expectation
- Law of Large numbers

$$f^* = \frac{bp - q}{b}$$

Explanation

- Now lets say, I tell you if 1 comes then I will give you 6 dollars otherwise you will lose 1 dollar
- 1= +6
- 2,3,4,5 or 6 = -1
- Should you play this game?

Explanation

- P=1/6
- Q= 5/6
- B= 6
- Kelly Fraction = [(1/6*3)-(5/6)]/3=1/18
- Good trade
- Positive expectations

$$f^* = \frac{bp - q}{b}$$

Explanation

- Did you spot the difference?
- When you are losing it will prevent you from increasing the stakes
- Will only let you bet when odds are favourable
- When you are winning increase your stakes
- When you are losing decrease your stakes

Games: favorable or unfavorable

- You can never win in the long run with a sequence of investments, all of which are unfavorable, using a mathematical scheme.
- Design strategy to create an edge.
- Make investments to achieve long run goal and manage risks
- Blend growth versus security to your risk tolerance and the situation at hand

	Edge	Probability of Winning
Blackjack	~1-2%	40-60%
Financial Futures	~10%+	2-98%
Horseracing	~10%+	20-98%
Lotteries	~50%+	less than 1%

Games: favorable or unfavorable

Success in investments has two key pillars:

- devising a strategy with positive expectation and
- betting the right amount to balance growth of one's fortune against the risk of losses.

A strategy which has wonderful asymptotic long run properties

- the log bettor will dominate other strategies with probability one ***and***
- accumulate unbounded amount more wealth.

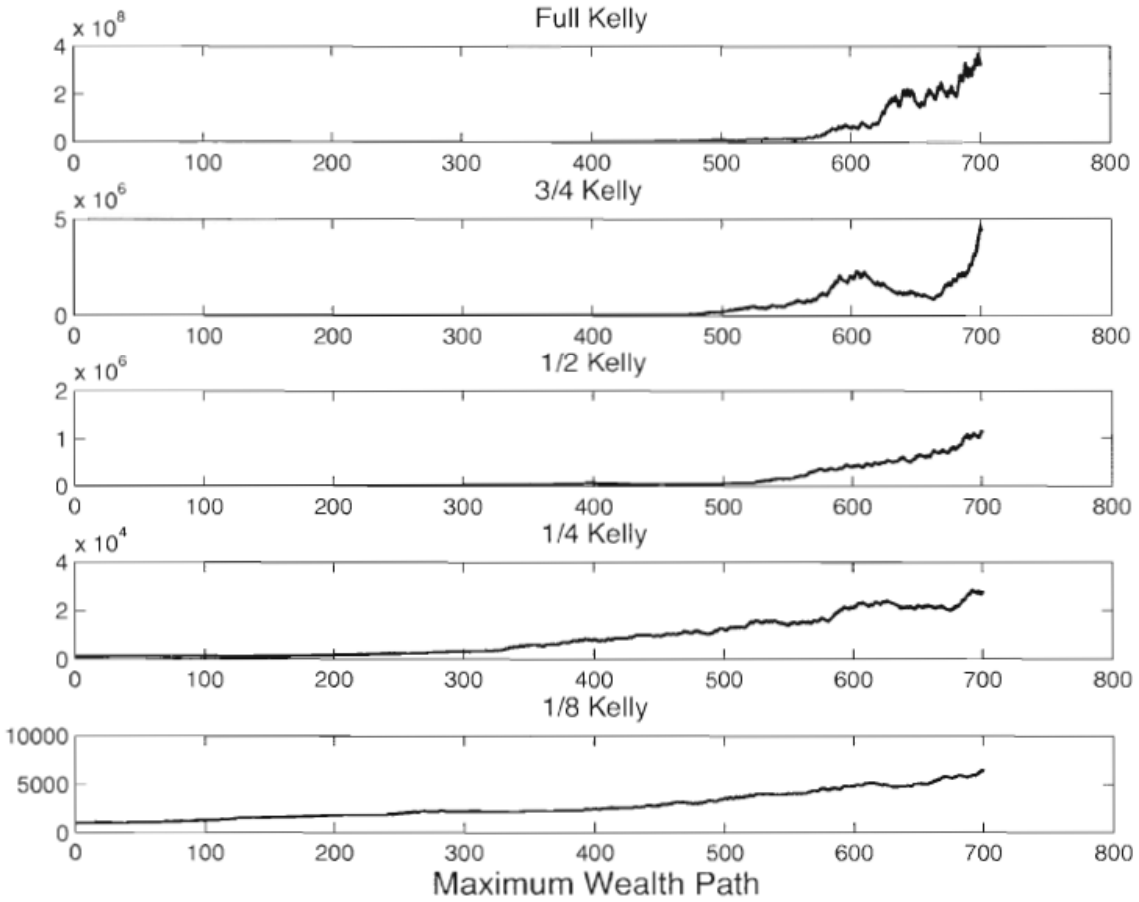
Fractional Kelly strategies provide more security but with less growth.

- William T. Ziemba worked/consulted with seven individuals who turned a humble beginning with essentially zero wealth into hundreds of millions (at least five are billionaires) using security market imperfections and anomalies in racing, futures trading and options mispricing.

- Once they reach 200-300 million, then **often** log --> linear: bet on anything with a "positive expectation" as long as you diversify and move their wealth into the best hedge and alternative investment funds

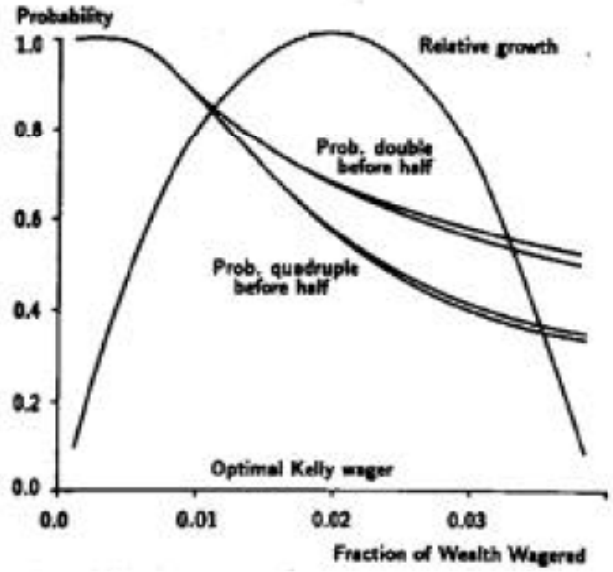
- **All** of them used Kelly or fractional Kelly betting strategies.

Why use Kelly?



Kelly strategy is good in long term

- In short run it can result in fluctuations in wealth
- Less risk taking investors can use half Kelly fraction



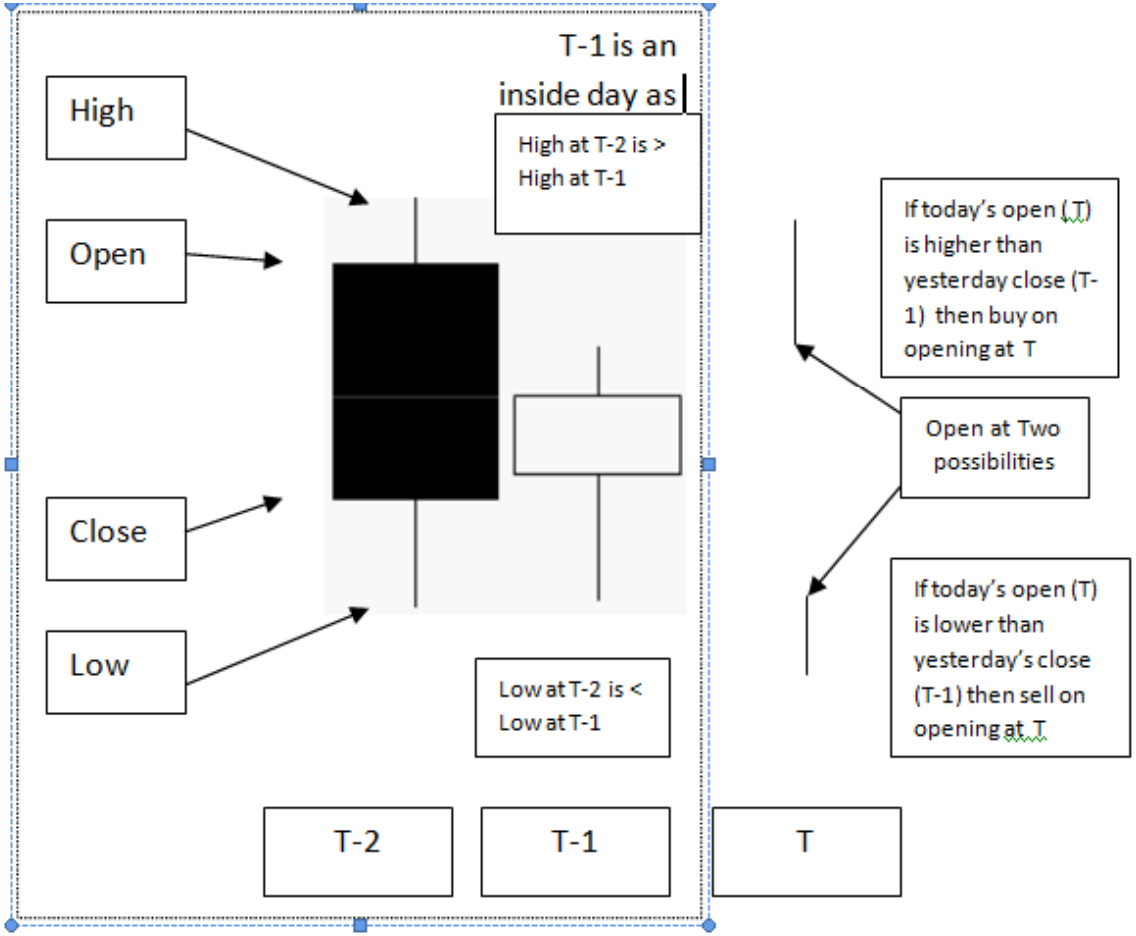
Good and bad properties of the Kelly criterion. Ziemba et al, Jan 1 2010

How does it relate to Buffet and other investors?

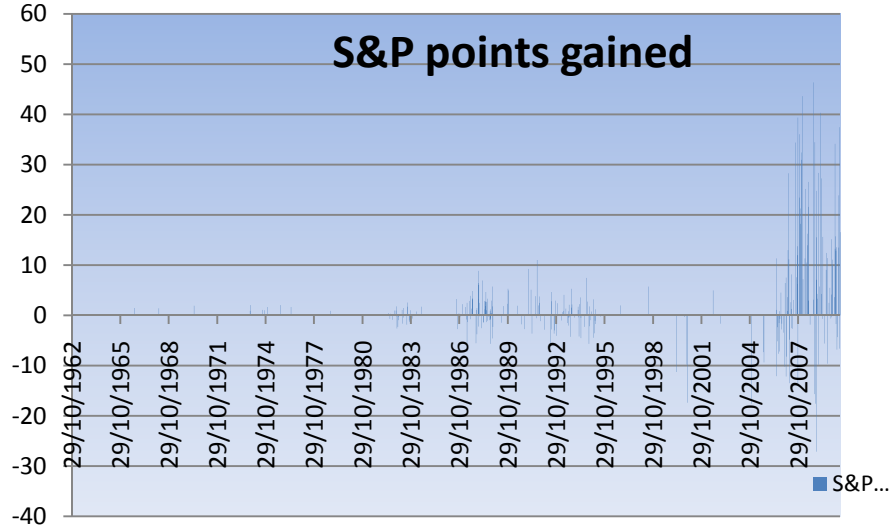
- When they are sure about something they go all in

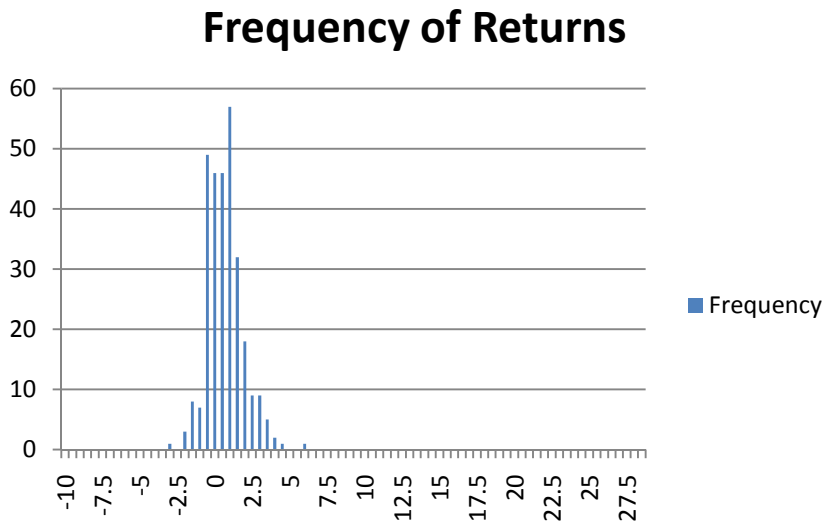
- They measure the risk-reward ratio

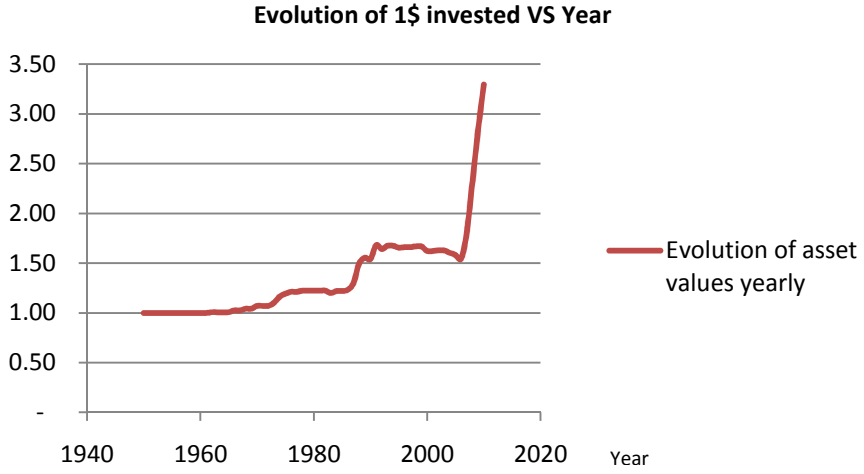
Practical Example



Practical Example







Number of winning Trades	180
Number of Loosing Trades	114
Probability of win	0.612245
Probability of losses	0.387755
average loss of S&P points	1.2716
average gain of S&P points	2.0577
Full Kelly	29.30%
Half Kelly	14.65%

Efficient Market Hypothesis?

- Impossible to predict prices of assets
  - Weak form
  - Semi-strong form
  - Strong form

- New research shows that certain degree of predictability of financial assets is required to compensate investors for risk

- New camp says it is not possible to generate excess return over return required to compensate investors for taking risk

Efficient Market Hypothesis?

- Harry Markowitz and traditional theory of Portfolio management fall into this camp
- Maximize arithmetic mean
- Arithmetic mean of 20 and 0= 20+0/2= 10

Efficient Market Hypothesis?

- Kelly bettors maximize geometric mean

- Geometric mean of 20 and 0= 0

- Geometric mean and arithmetic mean are equal when standard deviation is zero

- GM<= AM

Efficient Market Hypothesis?

- Kelly bettors = Wiki leaks
- Alternate concept of investing
- Stochastic Optimization

- Traditional Portfolio Theory= Traditional Media
- Assumes world is perfectly linear

Efficient Market Hypothesis?

$$r = R_f + \beta_3(K_m - R_f) + bs \cdot SMB + bv \cdot HML + \alpha$$

- R_f = risk-free return rate
- $K_{m=}$ is the return on the whole stock market
- β is analogous to the classical β but not equal to it, since there are now two additional factors to do some of the work
- *SMB* = small minus big (market capitalization)
- *HML* = high minus low ((book-to-price ratio)

Is alpha Generated?

Regression Statistics	
Multiple R	0.174081766
R Square	0.030304461
Adjusted R Square	0.02009714
Standard Error	1.144087246
Observations	289

	Coefficients(%, daily Values)	*Standard Error*	*t Stat*	*P-value*
Intercept	0.328246847	0.069388846	4.730542	3.53E-06
Mkt-RF	-0.04119252	0.055678344	-0.73983	0.460012
SMB	-0.197998308	0.118468165	-1.67132	0.095756
HML	0.288138905	0.12146654	2.372167	0.018348

LTCM

- What happens when you over bet?
- LTCM
- Founded by Nobel Prize Winner: Merton and Scholes
- Went Bust, Why?
- Leverage of 40:1, Over Betting
- Went against Kelly Criterion
- Historically a critique of Kelly Criterion

LTCM

- Invested their entire bankroll in what was low correlation products
- Collapse of Russia let to increasing correlation
- Increased stakes while taking positions

Conclusion

- Let the winners ride
- Shut the losers
- When you lose decrease your stakes
- When you win increase your stakes
- 95% of new traders or investors do the opposite
- Kelly bettor has a survival instinct
- Never bets his entire bankroll to insure against low chance of ruin and fat tails
- Kelly criterion is a mathematical proof that can be used intuitively

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